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## SOLUTIONS OF EXERCISES.

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### 320

A HOLLOW sphere, external and internal radii  $R$  and  $r$ , and specific gravity  $s$ , is partly filled with water, and floats in a pond, the water in the sphere being on a level with the surface of the pond. Find the quantity of water in the hollow sphere.

[*Artemas Martin.*]

SOLUTION.

Let  $h$  be depth of the water in the sphere; then  $h + R - r$  is the depth of sinking in the pond. If  $P, w, W$  be the weights of the water displaced, of the water in the sphere, and of the sphere respectively, then

$$P = w + W, \quad (1)$$

or

$$\pi(h + R - r)^2 [R - \frac{1}{3}(h + R - r)] = \pi h^2(r - \frac{1}{3}h) + \frac{4}{3}s\pi(R^3 - r^3).$$

This gives

$$h = \frac{4s(R^3 - r^3) + r(3R^2 - r^2) - 2R^3}{3(R^2 - r^2)} = \frac{4s(R^2 + Rr + r^2) - (2R + r)(R - r)}{3(R + r)};$$

whence the quantity of water in the sphere may be obtained from the expression  $\pi h^2(r - \frac{1}{3}h)$ .

[*W. O. Whitescarver.*]

### 324

A SUCTION pump is connected with a reservoir by a pipe of the same diameter as the piston. How many strokes are needed to bring water?

[*W. H. Echols.*]

SOLUTION.

Let the pipe be vertical, the lower side of the piston when at the upper end of the stroke be  $a$  feet above the surface of the reservoir, stroke of the piston  $b$  feet, the pressure of the atmosphere  $p$  = a column of water 34 feet high,  $\varphi_1$  the number of feet of water in the pipe at the end of the first stroke,  $\varphi_2$  at end of second stroke,  $\varphi_n$  at end of  $n$ th stroke.

Then will the  $a - b$  feet below the piston at the beginning of the first stroke be expanded to  $a - \varphi_1$  feet at the end of the stroke, resulting in a pressure of air

$$p_1 = \frac{a - b}{a - \varphi_1} p.$$

This pressure  $p_1$ , added to the weight of the water in the pipe, will equal an atmosphere ; hence

$$\frac{a-b}{a-\varphi_1} p + \frac{\varphi_1}{p} = p.$$

Similarly, at the end of the second stroke,

$$\frac{(a-b)(a-b-\varphi_1)}{(a-\varphi_1)(a-\varphi_2)} p + \frac{\varphi_2}{p} = p,$$

from which  $\varphi_2$  may be found. Finally, at the end of the  $n$ th stroke, we will have

$$\frac{(a-b)(a-b-\varphi_1)\dots(a-b-\varphi_{n-1})}{(a-\varphi_1)(a-\varphi_2)\dots(a-\varphi_n)} p + \frac{\varphi_n}{p} = p,$$

which will give  $\varphi_n$ . The number of strokes will be  $n$  when

$$\begin{aligned}\varphi_n &> a-b, \\ \varphi_{n-1} &< a-b.\end{aligned}\quad [De Volson Wood.]$$

### 327

FIND an approximate value for the perturbation of a comet by the sun, when the comet is very near a planet. (*See Watson's Astronomy*, p. 550).

#### SOLUTION.

Let  $M$  be the mass of the sun,  $r$  the distance of the sun from the planet,  $\Delta$  the distance of the comet from the planet, and  $\theta$  the angle between  $r$  and  $\Delta$ . Also let  $P$  be the disturbing force along  $\Delta$ , and  $T$  the disturbing force perpendicular to  $\Delta$  in the plane of the comet's orbit. Denoting by  $R$  the perturbative, or potential function, taking only the first term of this function, and neglecting the perturbation in latitude which is of the second order, we have

$$R = \frac{M\Delta^2}{2r^3} (1 - 3 \cos^2 \theta).$$

$$P = \frac{\partial R}{\partial \Delta} = \frac{M\Delta}{r^3} (1 - 3 \cos^2 \theta),$$

$$T = \frac{\partial R}{\partial \theta} = \frac{3M\Delta}{r^3} \cos \theta \sin \theta.$$

$$\therefore \text{Resultant} = \sqrt{P^2 + T^2} = \frac{M\Delta}{r^3} \sqrt{1 + 3 \cos^2 \theta}.$$

The expression in Watson's Astronomy is erroneous, as was pointed out by Professor Lehmann-Filhes. [A. Hall.]